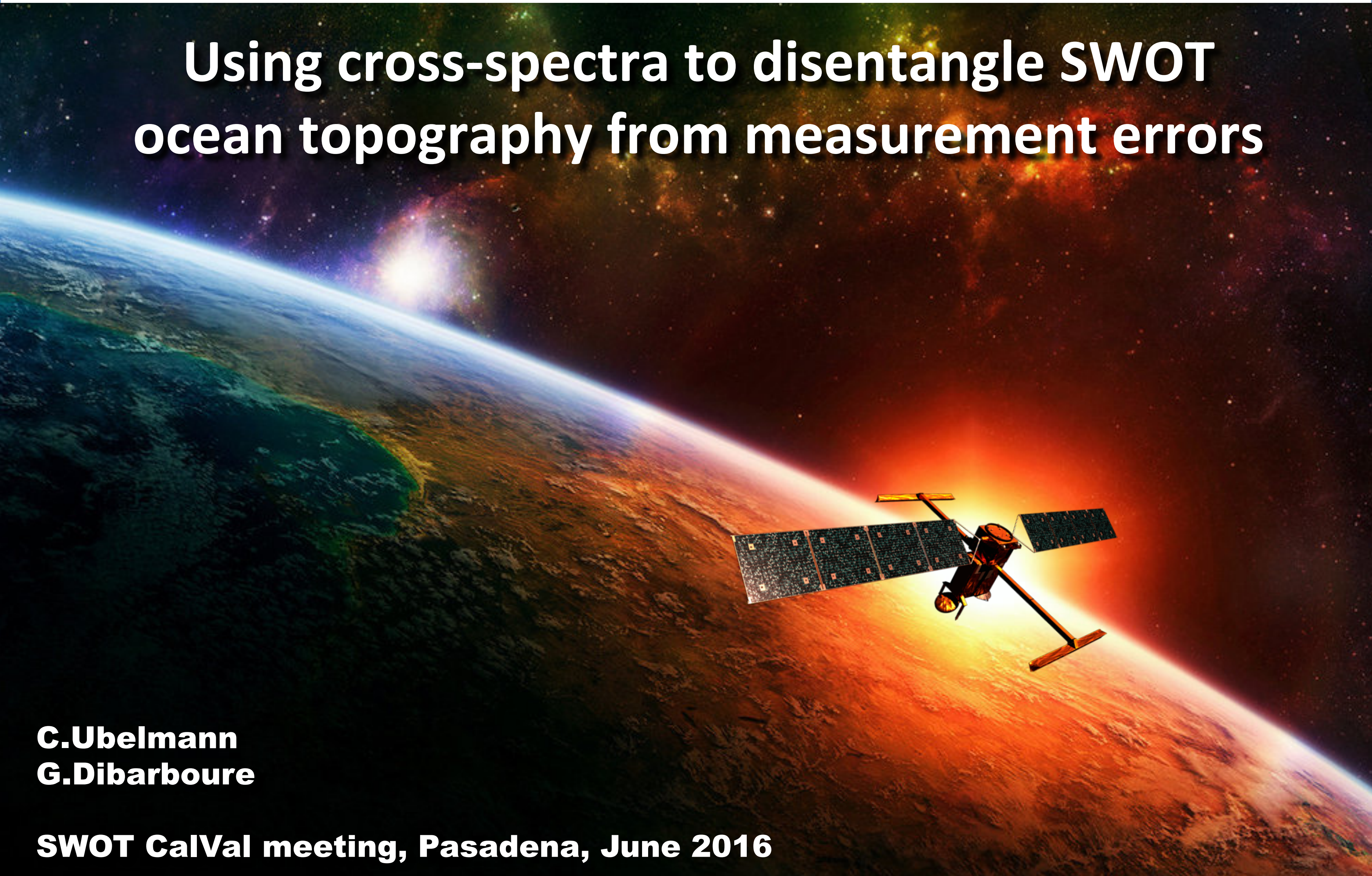
Using cross-spectra to disentangle SWOT ocean topography from measurement errors



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SWOT CalVal meeting, Pasadena, June 2016

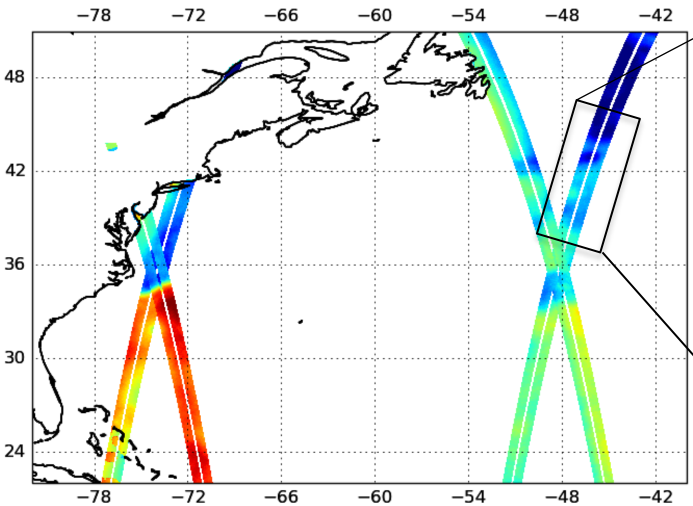
Introduction

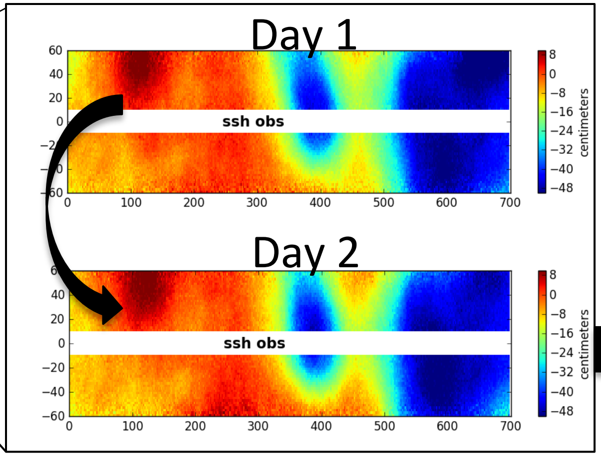
- Synoptic validation of the SWOT error spectrum is a hot topic
- Classical along-track 1D spectral analysis will measure the PSD of the <u>sum</u> of ocean signals plus all errors (not individual components)
- Measuring individual components is essential for Cal/Val and many applications (e.g. assimilation)

➡ Can we extract each spectrum from SWOT images with the sum of SSH + all errors ?

- In this study we use the <u>2D properties of KaRIN images</u> and the cross-track geometry of measurement errors to measure each component <u>separately</u>

Experimental setup with the SWOT Ocean simulator: dataset

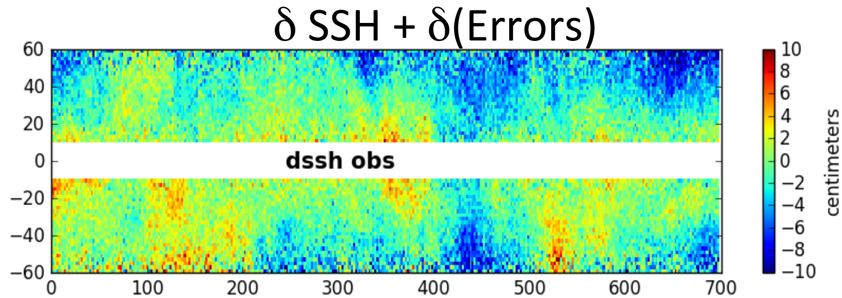




1-day difference (fast-sampling orbit)

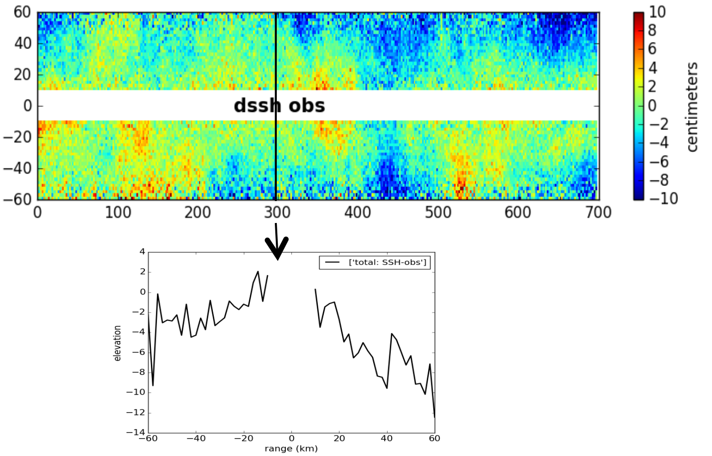
- 60-day worth of fast sampling phase simulation (MITgcm run)

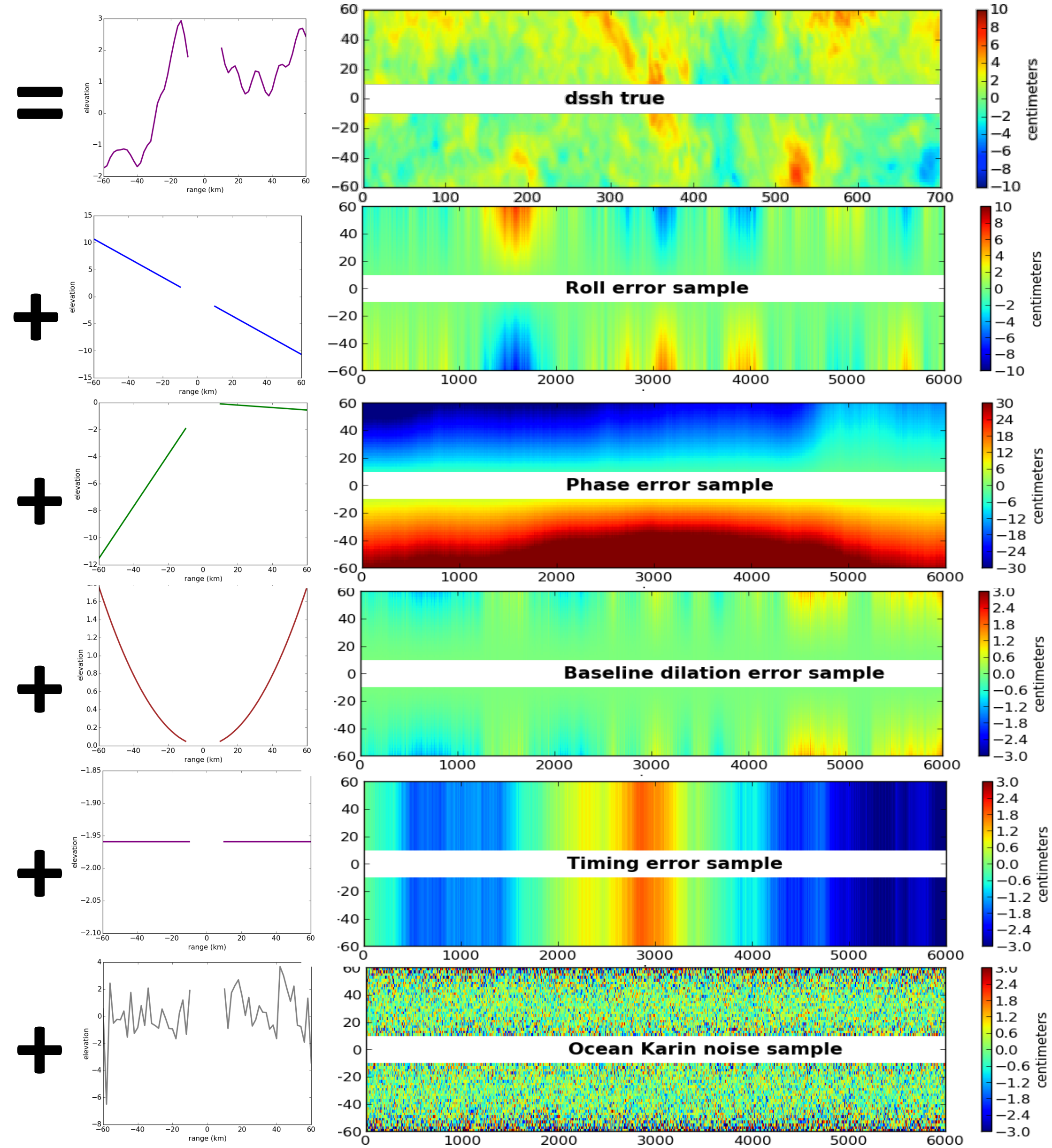
- 1-day differences cancel out a large fraction of the SSH signal



« Observed » inputs for analysing errors

What is measured in a 1-day difference ?





- Signal and errors are «visually» not separable in a single image

BUT

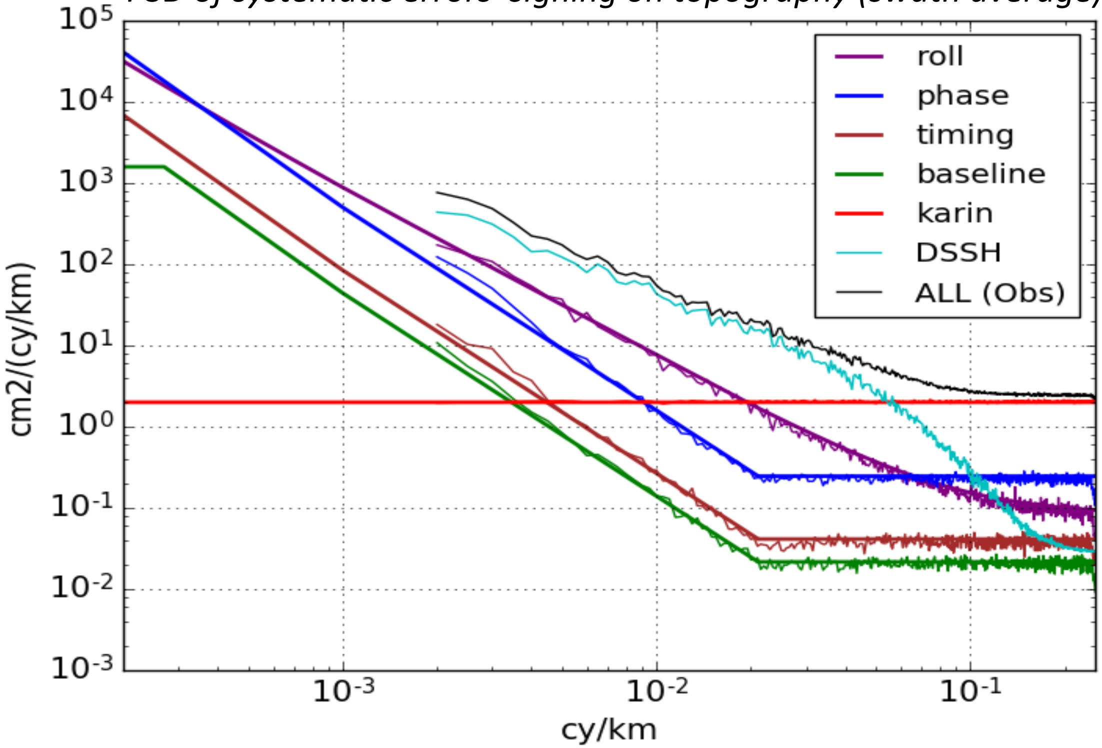
- Most error signals are range dependent (of known geometry)
- Ocean topography is not range-dependent

Error cross-track signatures should be statistically separable from topography

What is measured in a 1-day difference ? (cont.)

- One day difference measures the one-day variability plus twice the sum of all errors (we assume complete error decorrelation)

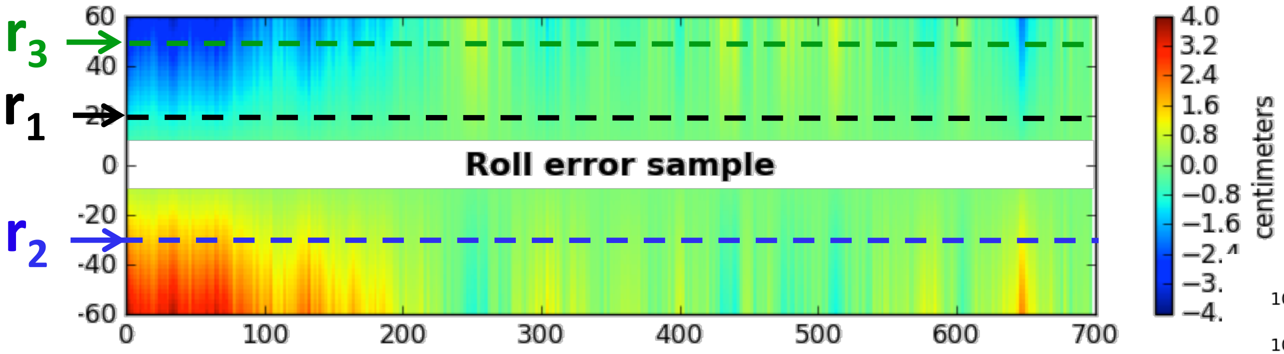
PSD of systematic errors signing on topography (swath average)



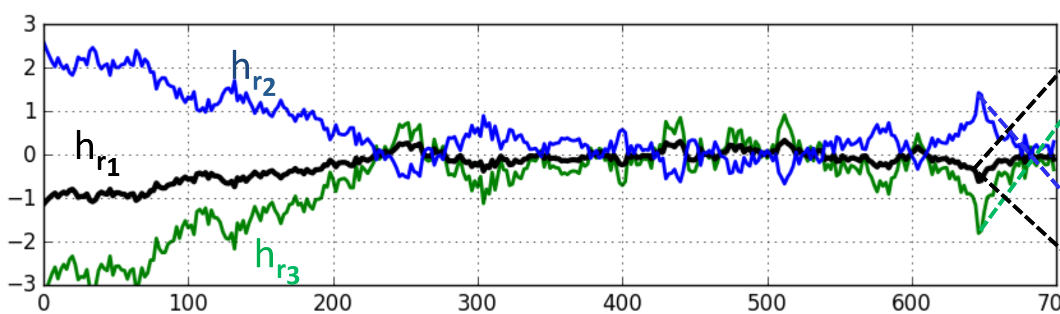
➡ Can we extract each spectrum from SWOT images with the sum of SSH + all errors ?

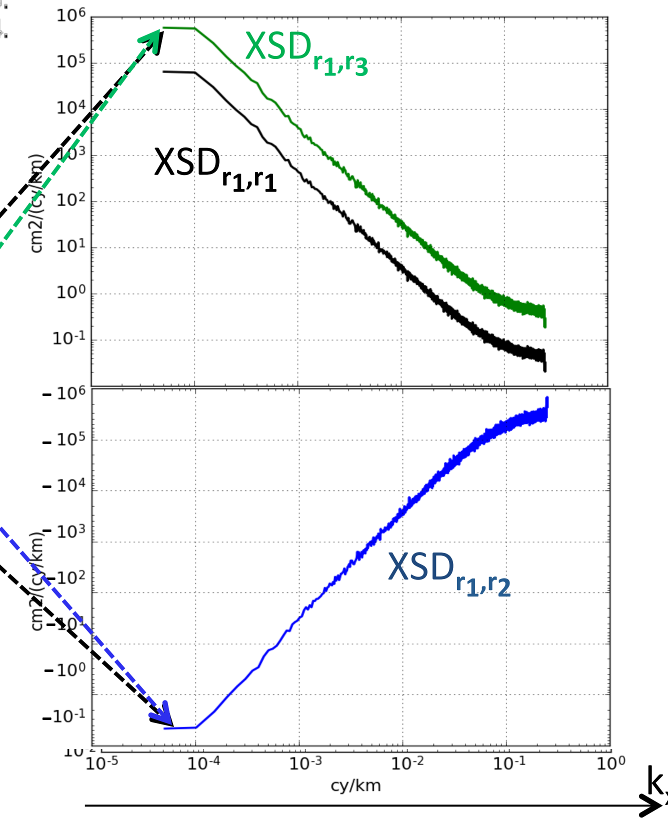
THE CROSS SPECTRAL SIGNATURE OF SWOT SIGNALS

Cross spectral density: example of roll



1D transect

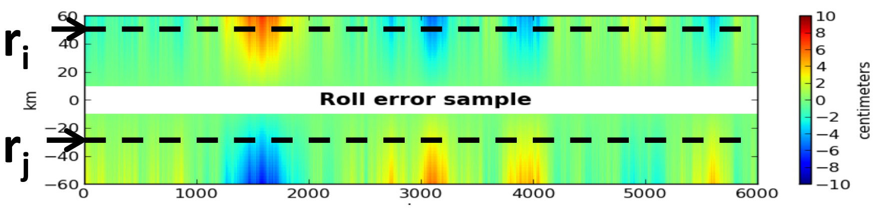


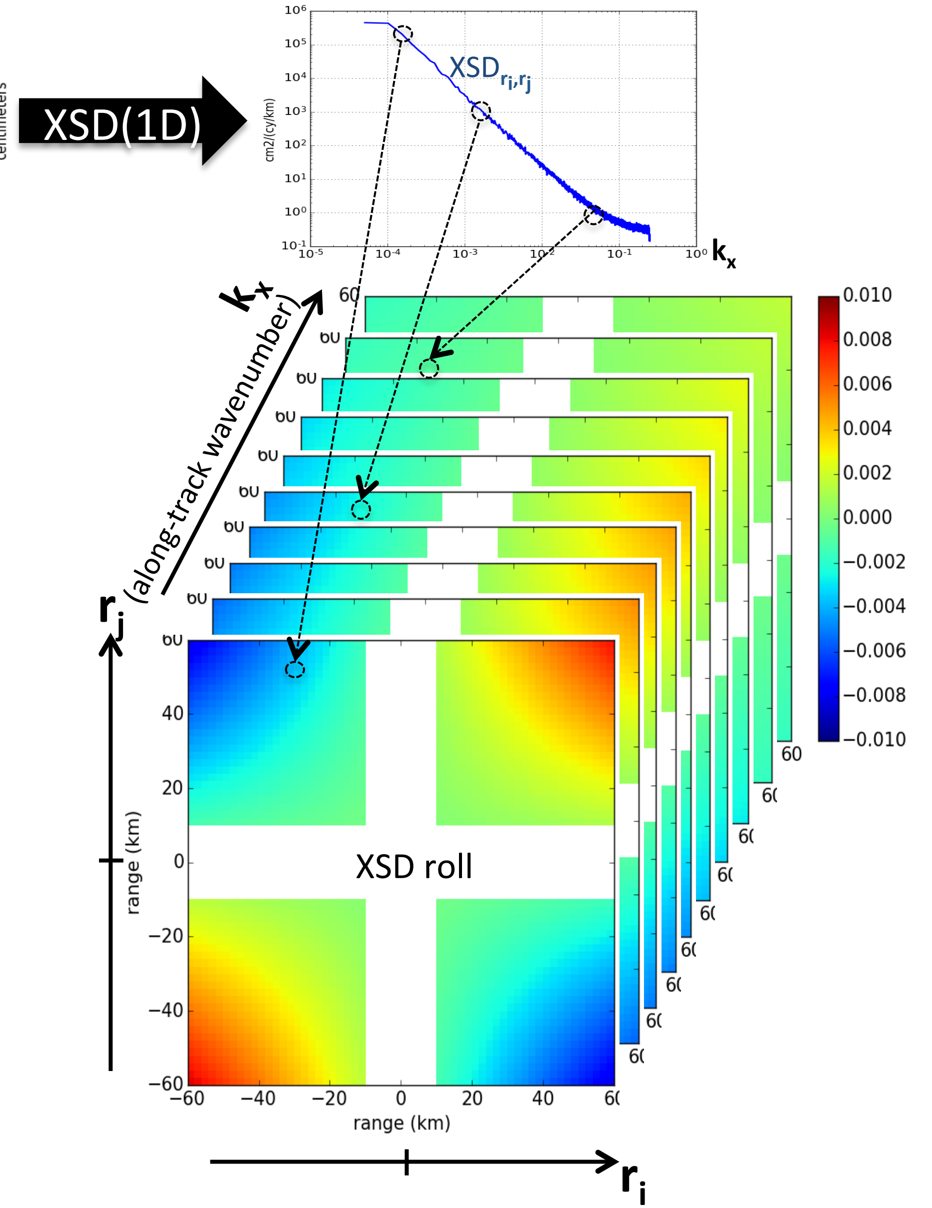


Cross spectral density (XSD):
$XSD_{r_i,r_j}=FFT(h_i).FFT(h_j)^*$

Along-track XSD

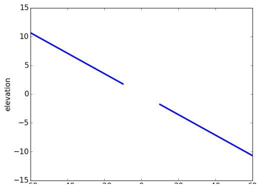
XSD cube: example of roll



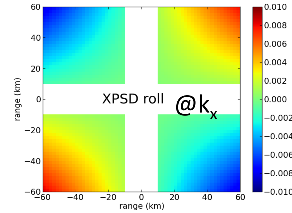


- For all r_i,r_j, insert the 1D function XSD(k_x) in a 3D cube
- **Yields a « XSD cube » which is a function of r_i, r_j and k_x**
- Here, roll has a specific 2D signature in each (r_i, r_j) slice

2D slice of XSD cube : signatures are specific

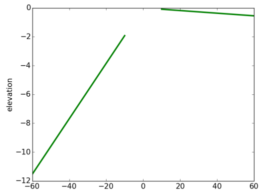


ROLL ERROR

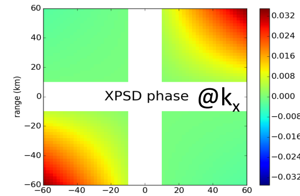


$XSD_{Hroll} = PSD_{roll} \cdot g_{roll}(r_i,r_j)$

with $g_{roll} = r_i\, r_j$

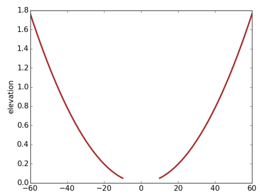


PHASE ERROR

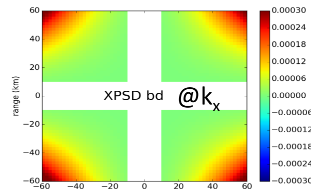


$XSD_{Hphase} = PSD_{phase} \cdot g_{phase}(r_i,r_j)$

with $g_{phase} = r_i\, r_j\, \delta(\mathrm{sign}(r_i),\mathrm{sign}(r_j))$

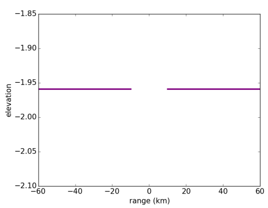


BASELINE ERROR

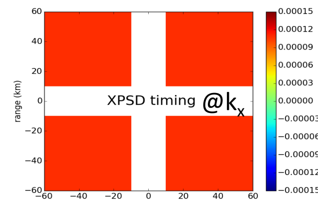


$XSD_{Hbd} = PSD_{bd} \cdot g_{bd}(r_i,r_j)$

with $g_{bd} = (r_i\, r_j)^2$

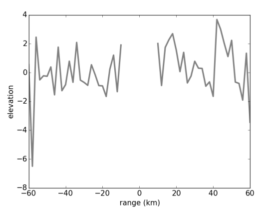


TIMING ERROR

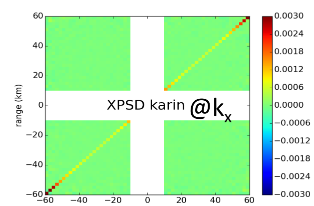


$XSD_{Htim} = PSD_{tim} \cdot g_{tim}(r_i,r_j)$

with $g_{tim} = 1$

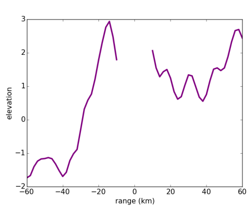


NOISE ERROR

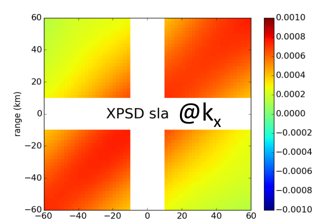


$XSD_{Hbd} = PSD_{noise} \cdot g_{noise}(r_i,r_j)$

with $g_{noise} = \delta(r_i,r_j)$



δSSH



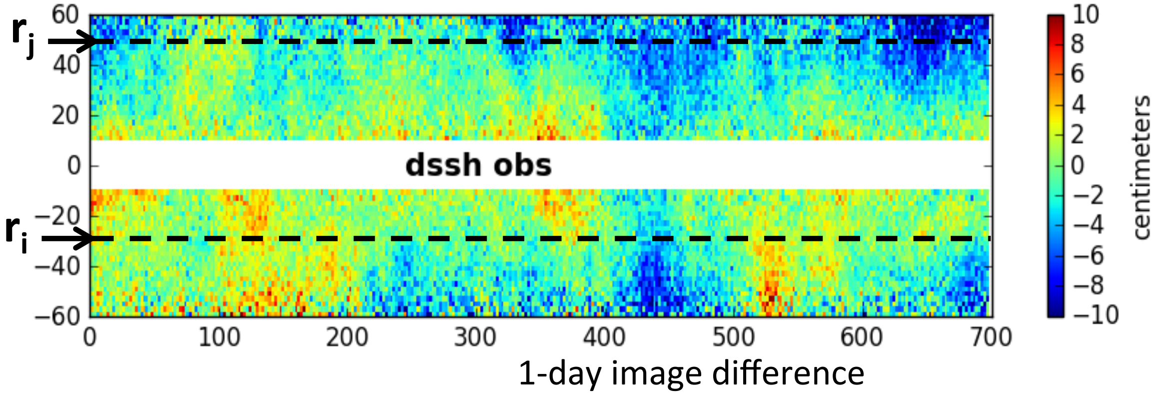
$XSD_{Hiso} = PSD_{iso} \cdot g_{iso}(r_i,r_j)$

with $g_{iso} = FFT^{-1}(PSD2_{iso}\,[k=k_x])[\delta r=|r_i - r_j|]\,]$

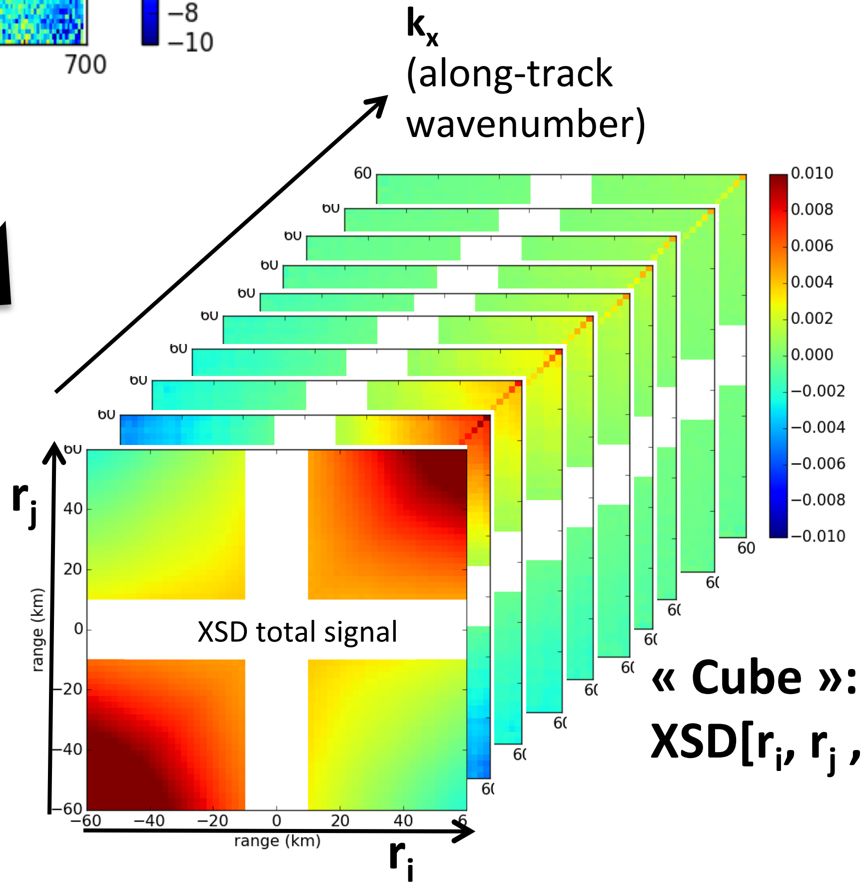
Details in backup

METHODOLOGY

Step 1: build the XSD cube from the observations

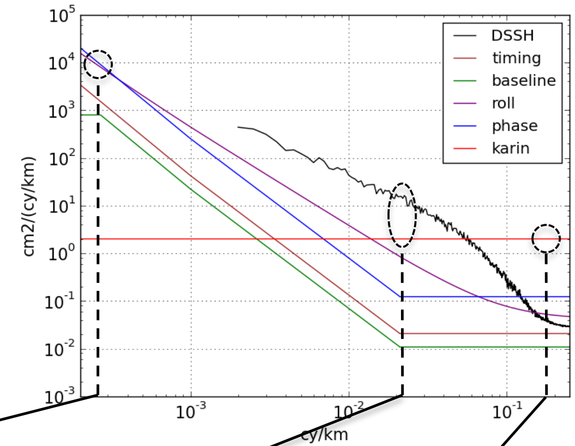


Build XSD cube of
δSSH + δ(Errors)

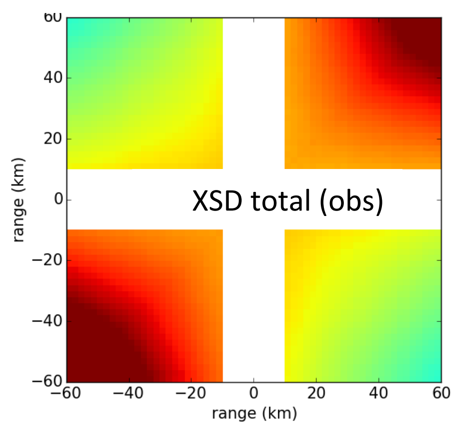


« Cube »: $XSD[r_i, r_j, k_x]$

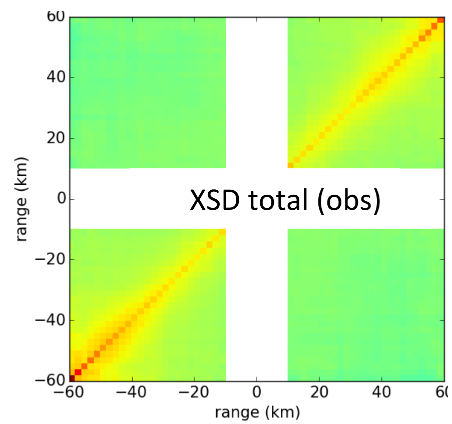
- XSD slices have very different patterns according to wavenumbers
- We recognize some combinations of analytical models



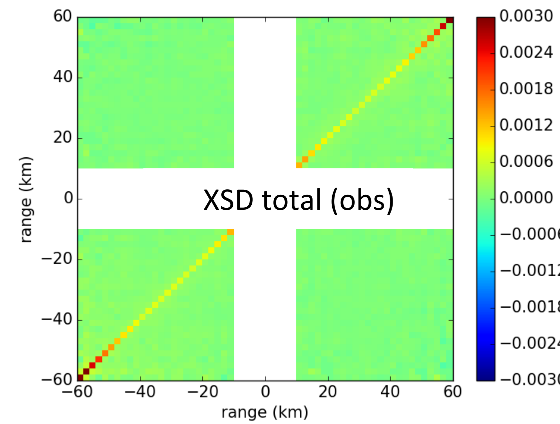
Long wavelengths: signature of roll and phase dominate



Medium wavelengths: signature of δSSH and noise dominate



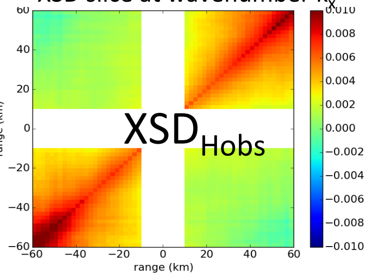
Short wavelengths: signature of Karin noise dominates



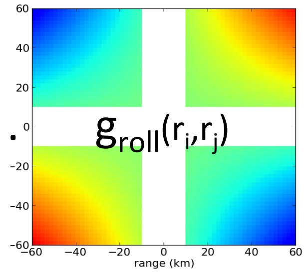
Step 2: Decompose the XSD slices

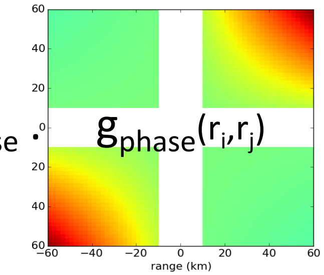
$$XSD_{Hobs} = XSD_{Hroll} + XSD_{Hphase} + XSD_{Hbd} + XSD_{Htiming} + XSD_{Hnoise} + XSD_{Hiso}$$

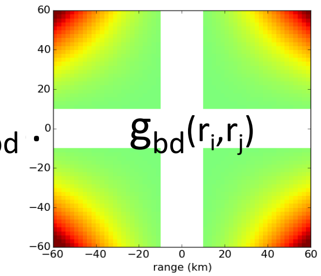
XSD slice at wavenumber k_x



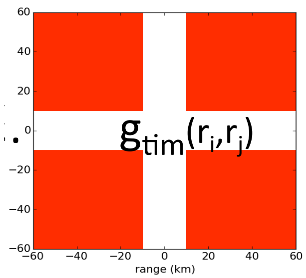
$$XSD_{Hobs} = PSD_{roll} \cdot g_{roll}(r_i,r_j) + PSD_{phase} \cdot g_{phase}(r_i,r_j) + PSD_{bd} \cdot g_{bd}(r_i,r_j)$$

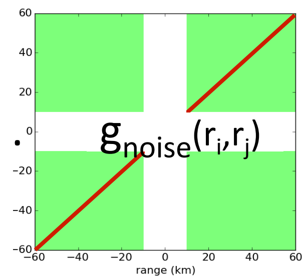


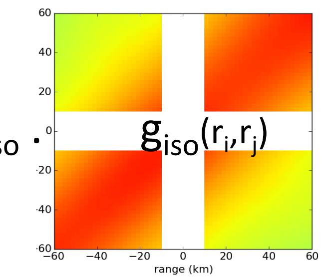




$$+ PSD_{tim} \cdot g_{tim}(r_i,r_j) + PSD_{noise} \cdot g_{noise}(r_i,r_j) + PSD_{iso} \cdot g_{iso}(r_i,r_j)$$







Obs

$$y = G \cdot \begin{pmatrix} PSD_{roll} \\ PSD_{phase} \\ PSD_{bd} \\ PSD_{timing} \\ PSD_{noise} \\ PSD_{iso} \end{pmatrix} = G \cdot x$$

det≠0

Least square inversion

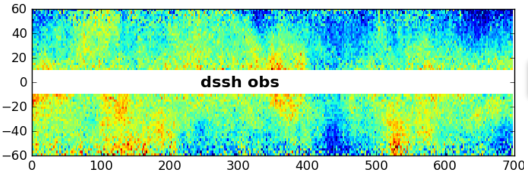
$$\mathbf{x_{est} = (G^T G)^{-1} G^T y}$$

Each PSD is estimated at the given wavenumber k_x

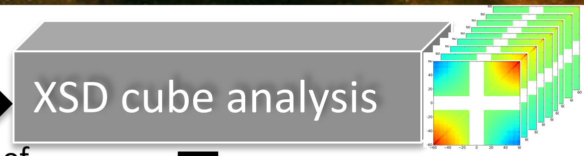
Then, we repeat for all k_x

RESULTS

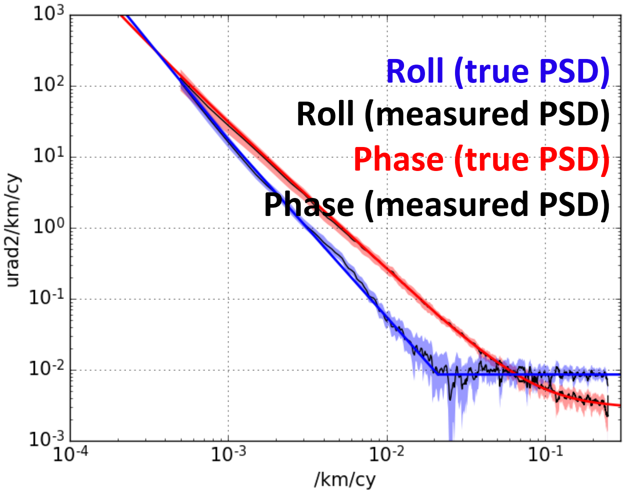
Effective reconstruction of individual spectra

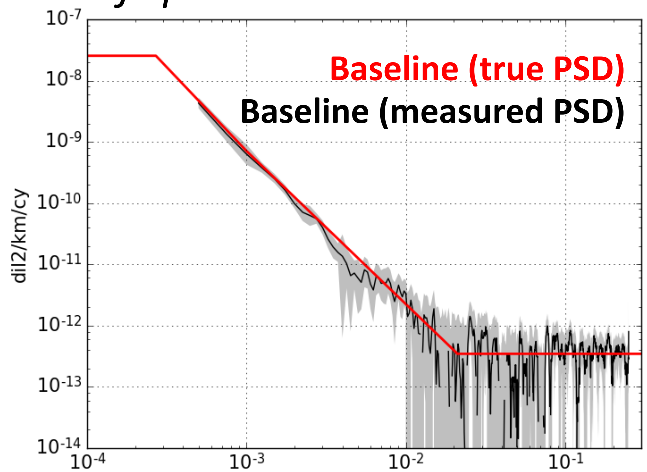


60-day worth of SWOT sample



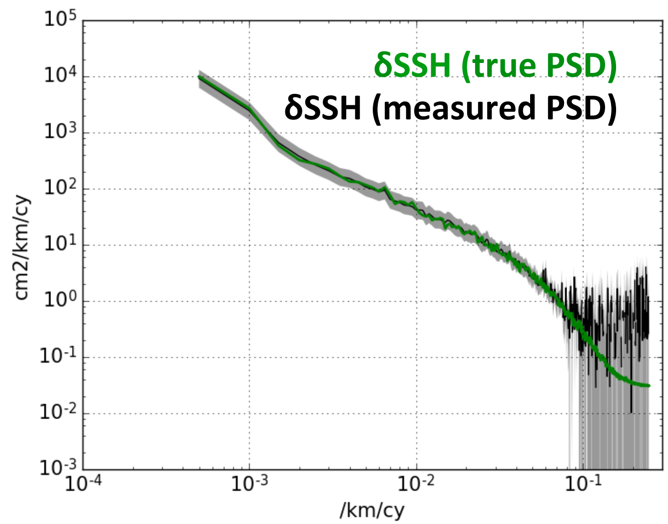
Breakdown of spectra



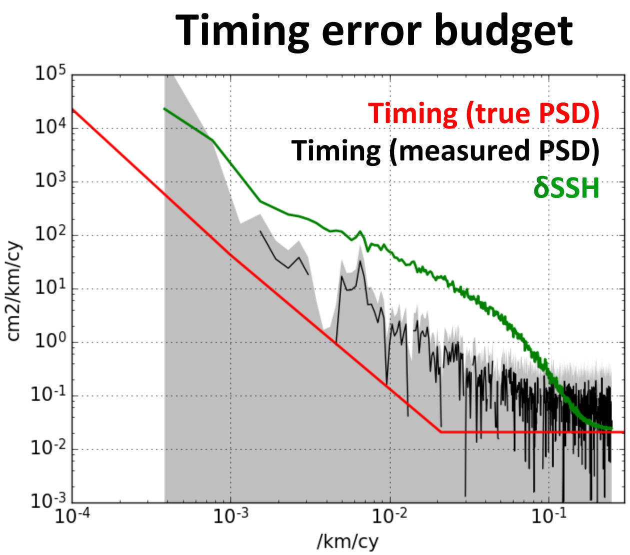


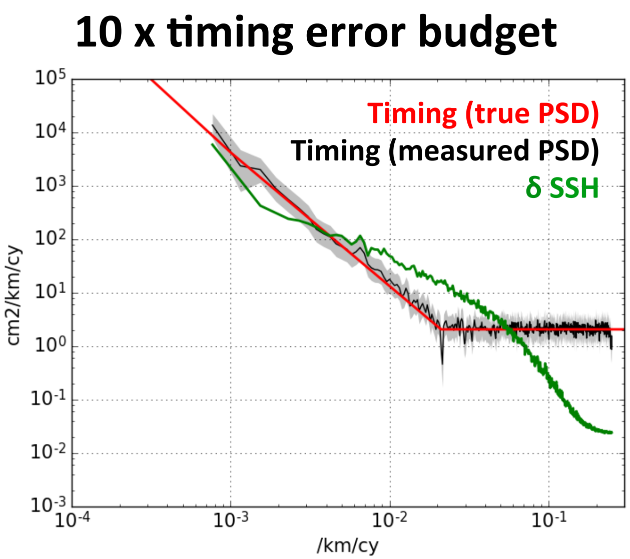
<u>Results after 60 days on 1-day orbit:</u>

- Estimation of mean PSDs works very well for dominent terms (roll and phase) at all wavelength.
- Even where their energy is >10 times less than δSSH
- δSSH well estimated, but difficult to separate it from Karin noise for λ<15 km



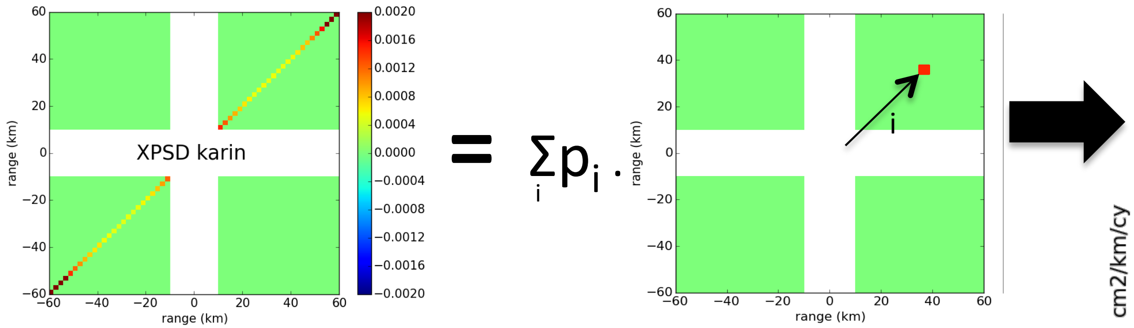
Effective reconstruction of individual spectra (cont.)

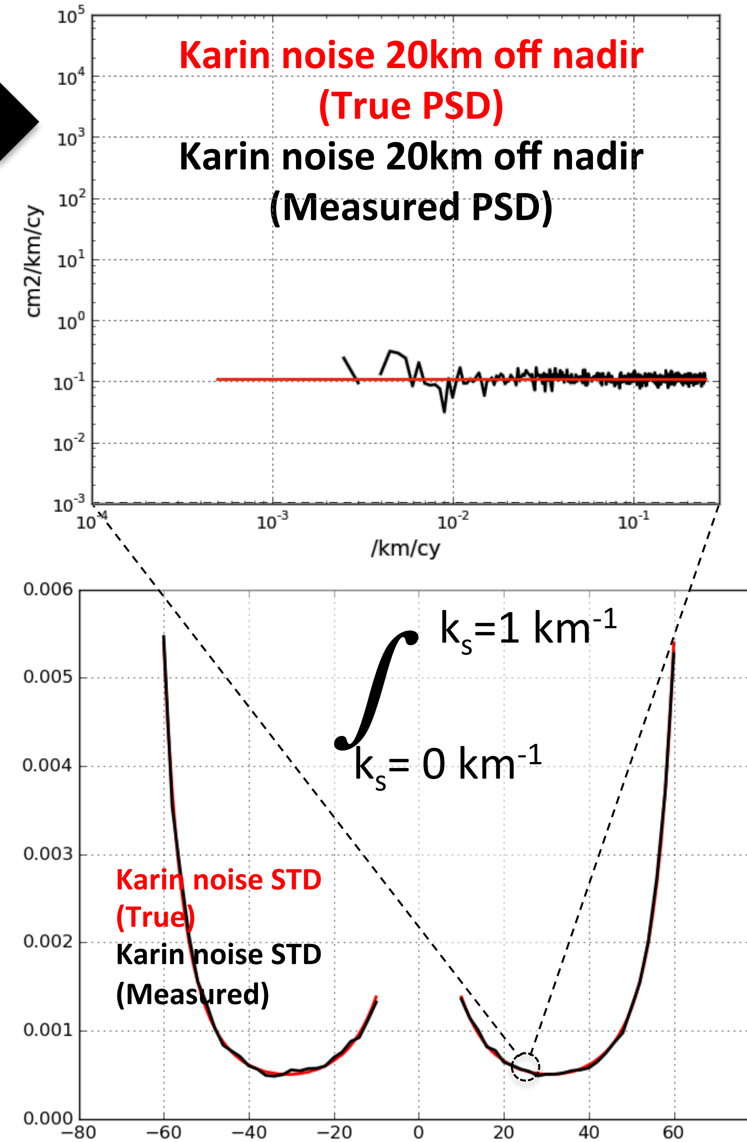




- The separation between timing and δSSH relies on δSSH isotropy assumption
- The timing range bias cannot be measured if it meets the requirement
  - In the error budget, this term is 100 times smaller than δSSH
  - Its XSD signature is difficult to separate from δSSH

- If the timing is larger than the requirements and become a threat for SSH, then it would be measured





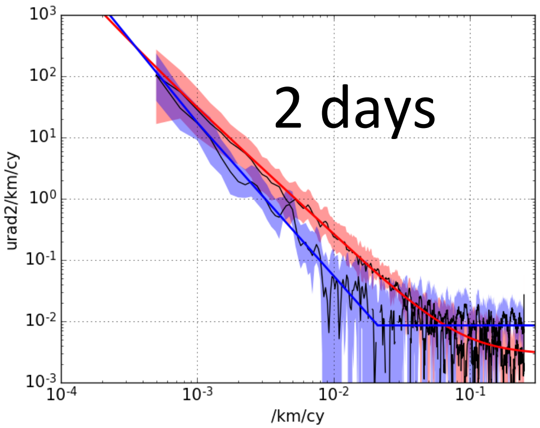
- Noise matrix can be also decomposed in the across-track (range) direction
- Each component describes the white noise spectrum for a given cross-track distance

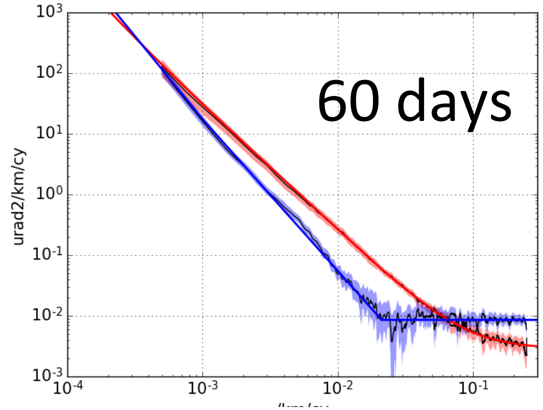
➔ Accurate estimation of the U-shaped noise specifications

- Noise is modulated by SWH and the science requirements are defined for SWH=2m ➔ possible to do this analysis in SWH bins in order to measure the SWH modulation of noise

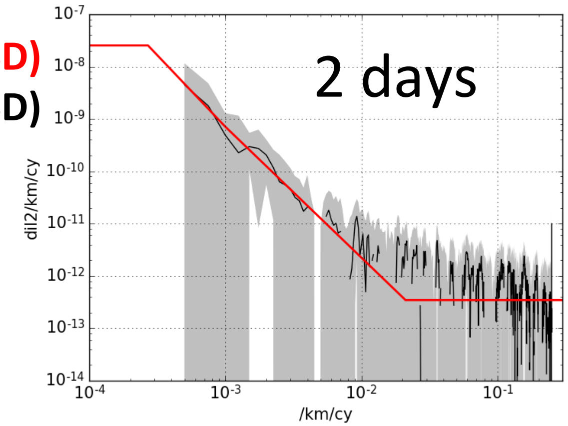
What if one PSD is time-varying?

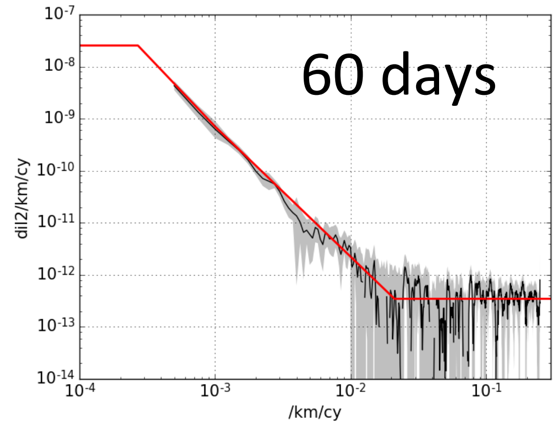
Roll (true PSD)
Roll (measured PSD)
Phase (true PSD)
Phase (measured PSD)





Baseline (true PSD)
Baseline (measured PSD)





- The method works on small datasets (here 5 days only)
  - The dominating PSDs are still accurate ➔ it is possible to infer rapid changes of these terms during the fast sampling phase
  - Difficult to measure frequently PSDs with little energy (here baseline)

Conclusions and outlook

Analysing the SWOT XSD cube works very well in simulations

- Measured PSD of roll, phase and baseline errors is very accurate (unique XSD signatures)
- White noise and its modulation by SWH and range is accurately measured
- DSSH spectrum is well estimated for wavelengths $\lambda > 15$ km (limit is noise-related)
- Timing range bias is more difficult to measure
    - If requirements are met, it is negligible w.r.t ocean topography
    - If requirements are not met (unlikely), it can be measured up to $\lambda = 200$ km

Why do we want to measure individual spectra?

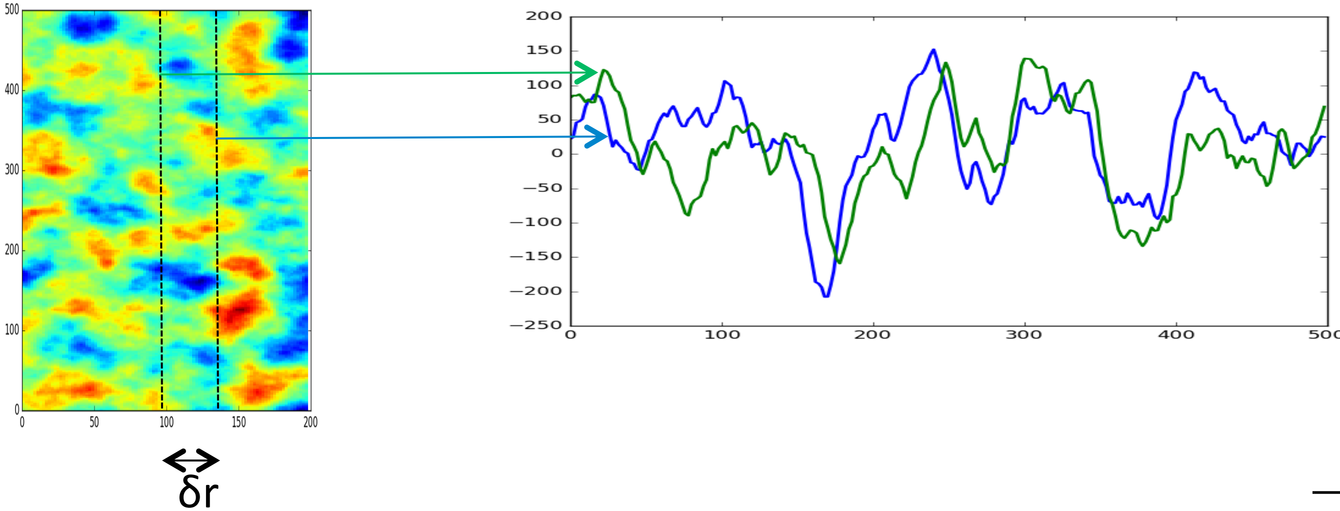
- Cal/Val during the fast sampling phase (to demonstrate that requirements are met)
- For all product usages that require spectral error description (e.g. assimilation or OI)
- XSD can be used on the 21-day orbit to detect changes in product accuracy

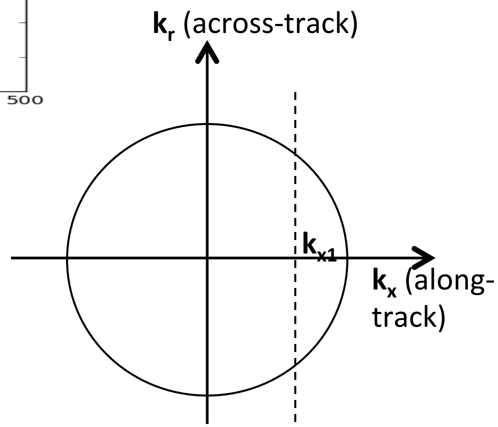
What if unexpected signatures are seen in flight data ?

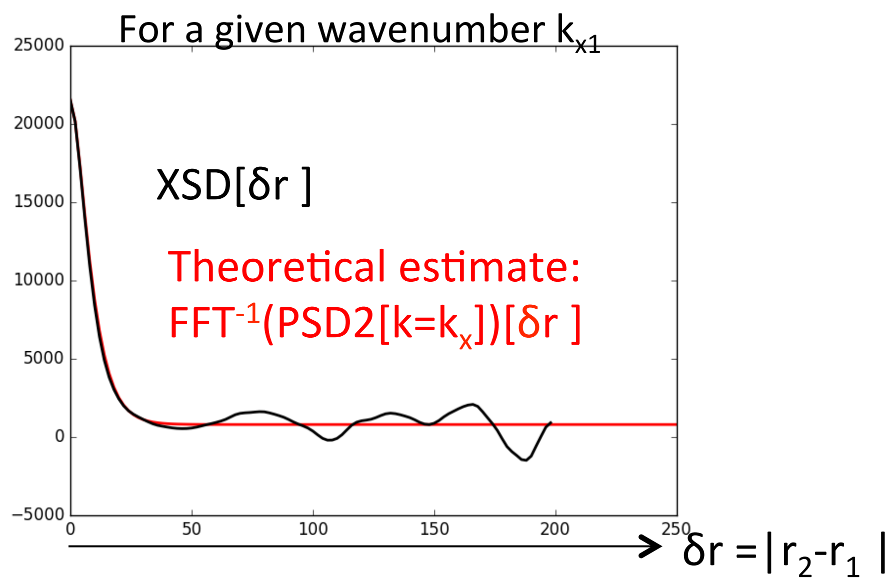
- SWOT XSD cube contains a wealth of information, certainly more than we can simulate today
- If unknown signatures are observed on the XSD cube, we can perform additional analyses to infer their origin (e.g. modulation by latitude, or H/V pol, or sea-state)

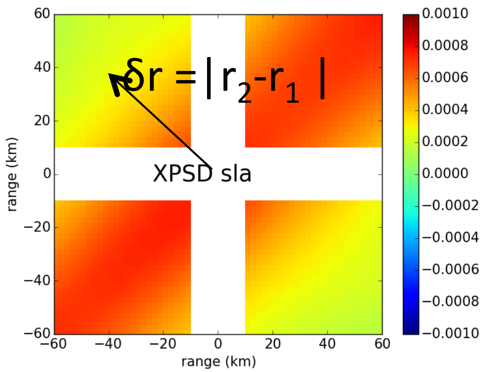
BACKUP SLIDES

XSD cube of isotropic signals



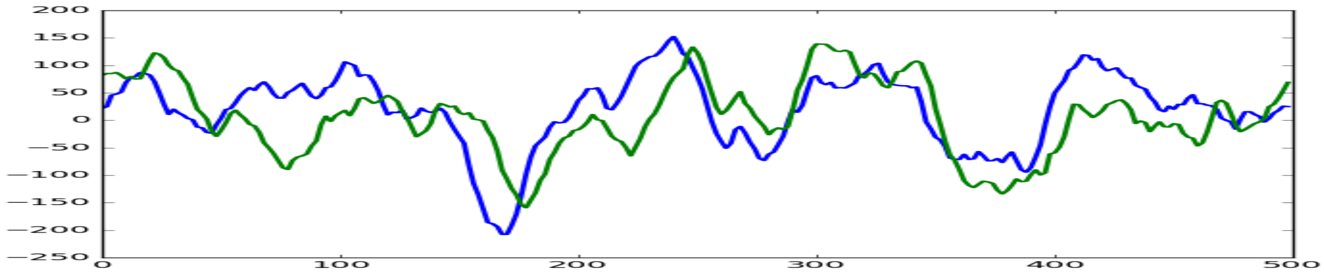


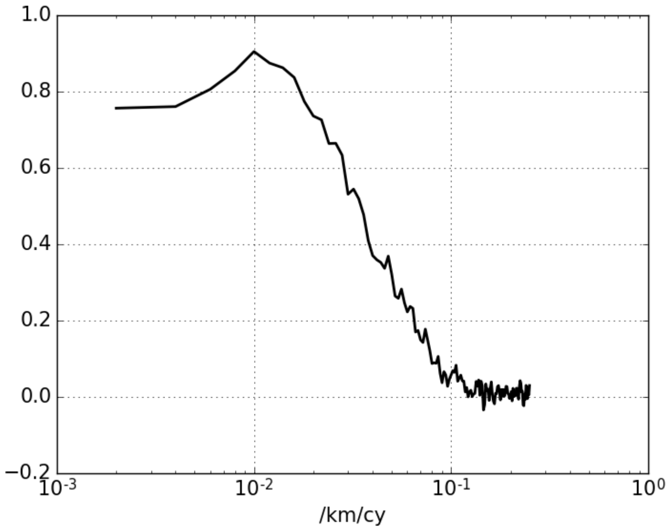




The Cross Spectral Density (XSD) between two series

Physical space (all integrated wavenumber)	Fourier space
Variance	Power Spectral Density (PSD)
Covariance	Cross Spectral Density (XSD)





- XSD is similar to a « covariance » at a given wavenumber. Can be negative
- XSD between two identical series is the PSD

$XSD_{h_1,h_2}=$